

Goldbach's conjecture satisfied with the strategy that the young Gauss invented to add the numbers from 1 to 100.

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1. Johann Friedrich Carl Gauss (4) was 9 years old when his teacher, to keep the whole class busy, asked his young students to add up the numbers contained in the number 100; Gauss, thanks to an intuition and a strategy, gave the correct answer in a few minutes. The young mathematician had visualized a symmetry that linked the numbers from 1 to 100 in fact, by superimposing the 100 numbers on two lines but in reverse order, we obtained 100 pairs of numbers whose sum was 101 and the result was the same for all pairs ($1 + 100$, $2 + 99$,, $99 + 2$, $100 + 1$). He excluded the pairs that had equal numbers and added up the remaining pairs which were half of the 100 pairs that were generated. Gauss obtained the answer to the question posed by the master by multiplying $50 \times 101 = 5050$. Today, to calculate the sum of the numbers contained in a number, we use the formula: $n(n + 1) / 2$.
- 1.1 Gauss, at the end of the 18th century, with the Fundamental Theorem of Arithmetic proved that positive integers greater than 1 are either prime numbers or are the product of prime numbers raised to a power n but Gauss, still young, showed that the same odd number, 101, is the sum of the even numbers with the odd numbers contained in an even number. The strategy invented by Gauss is indicative for satisfying Goldbach's conjecture
- 1.2 The numbers contained in the number "100 of Gauss" are 50 even numbers and 50 odd numbers. By overlapping the 50 even numbers on two lines and in reverse order, pairs of only even numbers are formed whose sum is equal to the even number 100 ($0 + 100$, $2 + 98$, ..., $50 + 50$, ..., $98 + 2$, $100 + 0$); by superimposing the 50 odd numbers on two lines and in reverse order, pairs of only odd numbers are formed whose sum is equal to the even number 100 ($1 + 99$, $3 + 97$, ..., $49 + 51$, $51 + 49$, ..., $97 + 3$, $99 + 1$); the two numbers of each pair are equidistant from the half of 100. Gauss's intuition "overlapping on two lines and in reverse order the numbers contained in one of the infinite even numbers", allows us to obtain that any even number is the result of two numbers as stated in the strong version of Goldbach's conjecture.
2. **Goldbach's conjecture** (2a) is one of the unsolved problems of number theory and is known to us in two versions. In 1742 Goldbach (2) affirmed that the infinite odd natural numbers, greater than 5, are the sum of 3 prime numbers, the conjecture thus formulated is known as the weak or ternary version of Goldbach's conjecture and, the mathematician not having proved it, sent it to his friend Euler (3) to prove it. Euler (3), aware of not being able to verify the infinite prime numbers, reformulated it in: all even numbers greater than 2 are the sum of two prime numbers; the conjecture thus formulated is known as the strong or binary version of Goldbach's conjecture.

The strong version of the conjecture, the one that refers to the infinite even numbers greater than 2 which are the result of the sum of two primes, has been verified by prof. Tomás Oliveira and Silva (5) who confirmed that even numbers less than or equal to 4×10^{18} are actually the result of the sum of

- two prime numbers but we can also affirm that since there is no prime number greater than all, the quantity considerable number of 4,000,000,000,000,000 verified even numbers or any quantity of verified even numbers is to be considered a primacy that will be surpassed by the even number which is only double the new and larger prime number which was not known but which Euclid and other mathematicians have shown that
- 2.1 **Euclid** ⁽¹⁾ by formulating $2 * n + 1$, with n denoting the productivity of all known primes, showed that we can generate ever larger odd numbers where they exist and new odd prime numbers that are ever larger are to be found of the first known ones of which to the production indicated with n , with which to generate an even number, the sum of two primes, greater than the equal sum of two known primes.
 - 2.2 **Euler** ⁽³⁾ states that the infinite even numbers are the sum of two prime numbers and as we can state that there is no prime number greater than all we can also state that there is no even number greater than all. Any known even number will not be double that prime number, the largest that Euclid generates and is that even, large, inaccessible and unreachable number which on the plane is the sum, $x_n + y_n$ where n is the same prime number that, even if it is not known, it exists on the oriented lines which are the abscissa and the ordinate of the Cartesian plane.
 - 2.3 **Gauss** ⁽⁴⁾, has shown that the infinite even numbers can be expressed as the product of prime numbers and this representation is unique if we ignore the order in which the factors appear but, as reported in point 1.1 above, he also showed that the even number 100 or one of the infinite even numbers is the sum of two of the numbers contained in the given number which, superimposed on two lines and in reverse order, generate all the pairs of even and odd numbers, $x_n + y_n$, whose sum is the even number.
The pairs $x_n + y_n = z$ that Gauss gets from an even number are pairs of numbers that are equidistant from half the even number; the numbers of these pairs whose sum is z is one of the infinite even numbers which: large, inaccessible and unreachable and is a number that is the result of two halves of z or of numbers equidistant from this half. Thales ⁽⁶⁾ affirms that it is possible to measure everything that can be indicated on the plane.
 - 2.4 **Thales** ⁽⁶⁾, Greek philosopher, scientist and mathematician, measured inaccessible and unattainable quantities (6a). The measurements that Thales made with the shadows or with the triangle that he generated on the sand of the shore, the "Cartesian" plane of 2600 years ago, can be redone on the current known Cartesian plane and, always considering that for the computability of the numbers we have sufficient space and time (4b), we can associate the infinite points of the plane to infinite even numbers $z = x_n + y_n$ sum of two primes or, to infinite odd numbers z sum of an even (which is the sum of 2 primes) plus a first.
 3. In one of the infinite even numbers there are odd prime numbers less than or equal to half of the even number and odd prime numbers greater than or equal to half of the even number; to satisfy Goldbach it is necessary to verify that the prime numbers, x_n , contained in the first half of the even number are not multiples of the primes less than or equal to the square root of the even

number and to verify that the prime number of the second half, y_n , is distant from the half of the even number how far away is the prime number x_n . To confirm the difficulties if not the impossibility of identifying measures with prime numbers, we know that the largest prime number known is a prime number that we identify as Mersenne number (12) because it is a power of $2^{82\,589\,933} - 1$, was found in 2018 by GIMPS (Great Internet Mersenne Prime Search) (13), takes up space and if you wanted to write it, you need 24,862. 048 decimal digits and it took time to find it and multiple computers connected to each other were used.

4. Gauss with the invention of couples, allows us to affirm that any even number is the result of the sum of two odd numbers contained in the even number but, with regard to the even number, the sum of odd primes allows us to state only that even numbers which are double of any prime number are the sum of two primes. Not knowing how many prime numbers are, what value they have and the distance that separates successive prime numbers, we cannot say that in all even numbers there are minor and major prime numbers and prime numbers equal to half the even number. The one who distinguishes and confirms the existence of minor prime numbers and prime numbers greater than half the even number is Joseph Bertrand (11) who in 1845 conjectured that between a number and its double, between n and $2n$, there is always a prime number. In 1850 Chebyshev (10) proved the conjecture that is known as Bertrand's Postulate and among the pairs that Gauss obtains from the numbers contained in an even number there will certainly be one made up of the two prime numbers of Bertrand's Postulate.
 - 4.1 Euclid with $2 * n + 1$ demonstrates that there is a new and larger prime number, Chebyshev demonstrates that between n and $2n$ there are at least two primes that form a pair of odd numbers that Gauss obtains from the odd numbers contained in an even number.
5. **The infinite even numbers** recalled by Euler in the strong or binary version of Goldbach's conjecture are equal to the result of the sum of pairs of numbers that Gauss forms with the numbers contained in the even number,
 - 5.1 they are equal: to the result of the sum of the one a pair of even prime numbers and to the sum of the pairs of odd prime numbers that Gauss forms with the numbers contained in the even number;
 - 5.2 are equal to the result of the sum of an odd prime number less than half of the even number and the odd prime that is always in its double
6. **The odd infinitives** recalled by Goldbach in the weak and or ternary version of the conjecture that bears his name are equal to the result of the sum of pairs of numbers that Gauss forms with the numbers contained in the even number
 - 6.1 they are equal to the result of the sum of an odd number and an even number, one smaller and the other greater than half the even number;

- 6.2 they are equal to the result of the sum of three primes, of which one is the odd number which is prime and the other two are the 2 primes whose sum is the even number;
7. **The odd infinities that Euclid generates** with the production of the first known ones to which he adds the unit, $2 * n + 1$
- 7.1 are equal to the result of the sum of an odd number and an even number one less and the other greater than half of the even number $2 * n$
- 7.2 they are equal to the result of the sum of three primes, of which one is the odd number prime and the other two are the 2 primes whose sum is the even number; one smaller and the other greater than half the even number
7. **The infinite numbers that are proved to be the product of prime numbers greater than or equal to 1 are also the sum of only two or only three primes.**

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